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The sums of the horizontal and vertical projections of AB, BC, CD give

$$8 \cos \alpha + 10 \cos \beta + 12 \cos \gamma = 20,$$

$$8 \sin \alpha + 10 \sin \beta - 12 \sin \gamma = 4.$$

These six equations furnish the theoretical solution of the problem.

The PROPOSER furnished a complete solution.

348 (Mechanics). Proposed by ALTON L. MILLER, Ann Arbor, Michigan.

If equilateral triangles be constructed on the sides of any triangle, their centers are the vertices of a new equilateral triangle. Show that the center of gravity of this new equilateral triangle coincides with the center of gravity of the original triangle.

SOLUTION BY EMMA M. GIBSON, Springfield, Mo.

Let ABC be the given triangle and let the coördinates of the vertices A, B, C referred to the rectangular axes ox and oy be (c, o), (o, b), (a, o), respectively. The equation of the line through (a, o) and (o, b) is

$$y = -\frac{b}{a}x + b. \tag{1}$$

The line from D, the third vertex of the equilateral triangle on BC, through $(a/2, b/2)$ and perpendicular to (1) is

$$y = \frac{a}{b}x + \frac{b^2 - a^2}{2b} \tag{2}$$

The line through C making an angle of 60° with (1) is

$$y = \frac{b + a\sqrt{3}}{b\sqrt{3} - a}(x - a). \tag{3}$$

Solving equations (2) and (3), the values of the coördinates of D are found to be $[(a + b\sqrt{3})/2, (b + a\sqrt{3})/2]$.

Similarly the coördinates of F and E are found to be $[(a + c)/2, \sqrt{3}(c - a)/2]$ and $[(c - b\sqrt{3})/2, (b - c\sqrt{3})/2]$, respectively.

Now the centers H, G, I of the three equilateral triangles are

$$\left(\frac{3a + b\sqrt{3}}{6}, \frac{3b + a\sqrt{3}}{6}\right), \quad \left(\frac{a + c}{2}, \frac{\sqrt{3}(c - a)}{6}\right), \quad \left(\frac{3c - b\sqrt{3}}{6}, \frac{3b - c\sqrt{3}}{6}\right),$$

respectively, since $\bar{x} = \frac{1}{3}(x_1 + x_2 + x_3)$, $\bar{y} = \frac{1}{3}(y_1 + y_2 + y_3)$. These points are the vertices of the new triangle and by the formula for the length of a line between two points, the three sides are proved equal. Hence, the new triangle is equilateral.

The coördinates of the center of gravity of the original triangle are $\bar{x} = \frac{1}{3}[a + c]$, $\bar{y} = \frac{1}{3}b$. The coördinates of the center of gravity of the new triangle are

$$\bar{x} = \frac{1}{3}\left[\frac{a + b\sqrt{3}}{2} + \frac{a + c}{2} + \frac{c - b\sqrt{3}}{2}\right] = \tfrac{1}{3}(a + c),$$

$$\bar{y} = \frac{1}{3}\left[\frac{b + a\sqrt{3}}{2} + \frac{\sqrt{3}(c - a)}{2} + \frac{b - c\sqrt{3}}{2}\right] = \tfrac{1}{3}b,$$

which are the same as those obtained for the original triangle.

Also solved by HORACE OLSON and ROGER JOHNSON.

268 (Number Theory). Proposed by FRANK IRWIN, University of California.

Show that in any arithmetical progression, whose first term a_1 and common difference d are positive integers, any required number of consecutive terms may be found, no one of which is a prime number.

Solution by B. F. Yanney, College of Wooster, Ohio.

Suppose that the theorem is not true, and that n is the greatest number of consecutive terms in the progression no one of which is a prime.

Consider any $n + 1$ consecutive terms of the progression, as

$$A_1 = a_1 + kd,\ A_2 = a_1 + (k+1)d,\ A_3 = a_1 + (k+2)d,\ \cdots,\ A_{n+1} = a_1 + (k+n)d.$$

Set $M = A_1A_2A_3 \cdots A_{n+1}$. Then will $A_1' = A_1(M+1)$, $A_2' = A_1(M+1) + d$, $\cdots$, $A_{n+1}' = A_1(M+1) + nd$ be $n+1$ consecutive terms of the progression, no one of which is a prime. For consider any one of them, as

$$A_{r+1}' = A_1(M+1) + rd = (a_1 + kd)(M+1) + rd = (a_1 + kd + rd)(M+1) - rdM,$$

which is evidently not prime, since M is a multiple of $a_1 + kd + rd$. We are thus led to a contradiction. Hence the denial of the theorem must be withdrawn, and the theorem is true.

Also solved by H. N. Carleton. Elijah Swift, Horace Olson and Louis Clark.

269 (Number Theory). Proposed by ARTEMAS MARTIN, Washington, D. C.

Find three rectangular parallelepipedons whose edges are rational whole numbers, and whose solid diagonals are equal, and rational whole numbers.

I. Solution by the Proposer.

Let w, x and y denote the lengths of the edges, and z the solid diagonal, of any one of the three required solids; then we must have

$$z^2 = w^2 + x^2 + y^2.$$

Put $x = np$, $y = nq$, $z = w + nr$; then

$$(w + nr)^2 = w^2 + (np)^2 + (ny)^2 = w^2 + 2nrw + n^2r^2,$$

which gives, after dividing by n,

$$w = \frac{n(p^2 + q^2 - r^2)}{2r}, \quad \text{and} \quad z = \frac{n(p^2 + q^2 + r^2)}{2r}.$$

Now take $n = 2r$ and we get the integral values

$$z = p^2 + q^2 + r^2, \quad w = p^2 + q^2 - r^2, \quad x = 2pr, \quad y = 2qr,$$

for one of the solids. The other two solids are obtained by interchanging the values of p, q, r in the expressions for w, x, and y.

Hence

$$\begin{aligned}(p^2 + q^2 + r^2)^2 &= (p^2 + q^2 - r^2)^2 + (2pr)^2 + (2qr)^2 \\ &= (p^2 + r^2 - q^2)^2 + (2pq)^2 + (2qr)^2 = (q^2 + r^2 - p^2)^2 + (2pq)^2 + (2pr)^2.\end{aligned}$$

Take $p = 4$, $q = 2$, $r = 1$; then the solids are

21, 19, 8, 4; 21, 16, 13, 4; 21, 16, 11, 8 (diagonals 21).

Take $p = 4$, $q = 3$, $r = 2$; then they are

29, 21, 16, 12; 29, 24, 12, 11; 29, 24, 16, 3 (diagonals 29).

The values of p, q, r may be chosen at pleasure.

II. Solution by C. F. Gummer, Queen's University, Kingston.

We have to find three solutions of the Diophantine equation,

$$r^2 - z^2 = x^2 + y^2, \tag{1}$$

having the value of r in common.